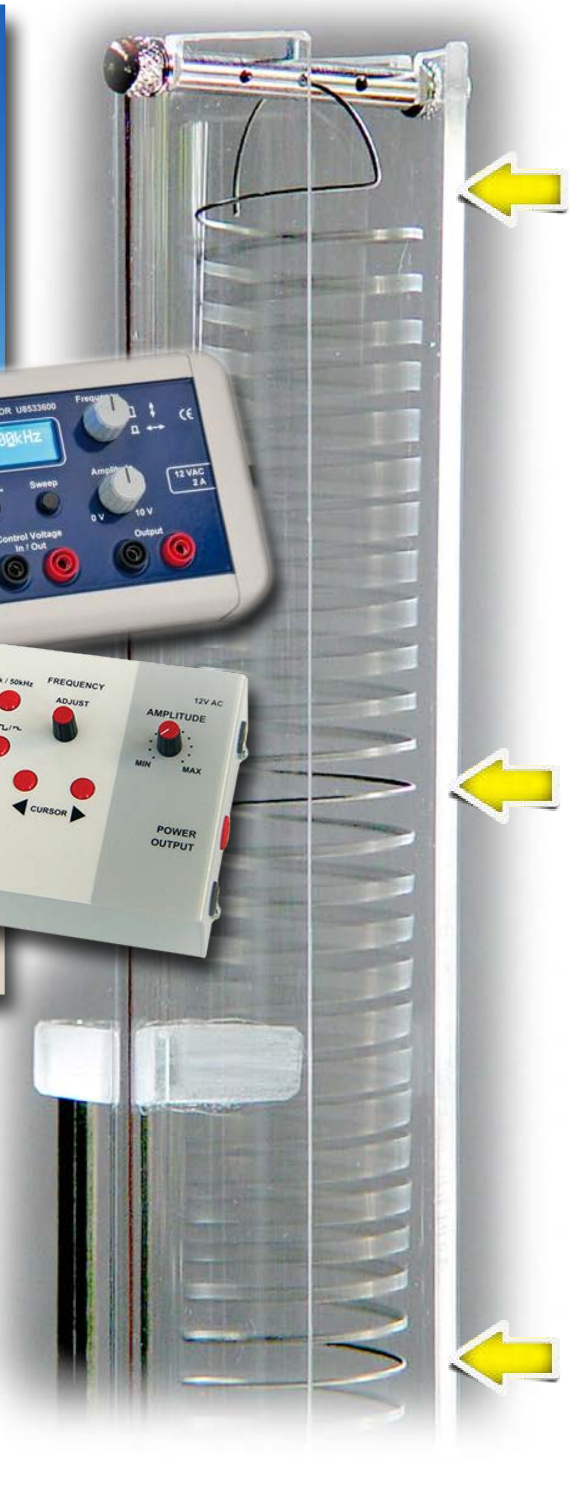
Oscillation and Wave Apparatus



Experimental Guide
Version 1.1

Oscillation and Wave Apparatus

The development of the apparatus is based on the well-known Prytz Oscillating Apparatus. The components are a Plexi-glass tube, inside which a helical spring is hanging from the top, with a bowl loaded with a circular magnet. Surrounding the tube, a vertically adjustable circular induction coil that connects to for example the *Function Generator F12 or F50*.

In General: With the induction coil connected to the function generator, the helical spring pendulum is set to vibrate - longitudinal oscillations - that moves up through the spring and is reflected from the fixed point of the spring at the top. The spring-particles are now influenced by both the incoming and the reflected wave. For specific drive frequencies a resonance case builds up. This is recognized as a standing wave on the spring in form of easy recognizable distribution of knots and bows - contraction and expanding of the spring windings.

On the base of the apparatus there are three *adjustment screws*. When the spring-pendulum is fixed with the top bar on the tube, the adjustment screws are set, so that the spring-pendulum with bowl and magnet does not touch the tube.
The spring pendulum must oscillate free from the tube.

The *Function Generator F12* is especially suited for delivering the **drive frequencies** - easy to adjust with respect to frequency and amplitude - and the most important: the frequency with a precision in adjusting and reading, that is a must in exploring the intentions that led to the construction of the *Oscillation- and Wave apparatus*.

The Oscillation and Wave apparatus is thought to show the finer details of a loaded spring as carrier and longitudinal waves and their interference - standing waves -

In the theory the 4 describing **apparatus-sizes** are used:

Spring constant = k
Spring mass = m
Spring length = L
that is windings total length under load
Load (bowl + weight) = M

and the 3 **wave parameters**:

Wavelength λ
Cyclic frequency ω
Velocity v
In equation:

$$\omega = \frac{2\pi}{T} = 2\pi f = v\left(\frac{2\pi}{\lambda}\right)$$

The more exact the **parameters of the apparatus** are known - the better results comparing the experimental measurements and the refined wave-theory for the resonances.

The **theory** of the resonances of the spring results in the following equation for calculation of the resonance frequencies f and the wave speed v

$$\cot\left(2\pi f\sqrt{\frac{m}{k}}\right) = \left(\frac{M}{m}\right)\left(2\pi f\sqrt{\frac{m}{k}}\right)$$

$$v = L\sqrt{\frac{k}{m}}$$

The transcendent equation is only in two border cases easy to solve:

1) M is large compared to the mass of the spring

that is the spring is fixed at both ends, then $\tan\left(2\pi f\sqrt{\frac{m}{k}}\right) = 0$ <=>

$$2\pi f\sqrt{\frac{k}{m}} = p\pi \quad \text{with p = 1, 2, 3, 4, ...} \quad \Leftrightarrow$$

$$f = \left(\frac{n}{4}\right)\sqrt{\frac{k}{m}} \quad \text{with n = 2, 4, 6, ...}$$

2) M = 0 if the spring is free, as

$$\cot\left(2\pi f\sqrt{\frac{m}{k}}\right) = 0 \quad \Leftrightarrow$$

$$2\pi f\sqrt{\frac{m}{k}} = \frac{\pi}{2} + p\pi \quad \text{with p = 0, 1, 2, 3, ...} \quad \Leftrightarrow$$

$$f = \left(\frac{n}{4}\right)\sqrt{\frac{k}{m}} \quad \text{with n = 1, 3, 5, 7, ...}$$

Experiments

1. Experiment

Resonance curve for the basic oscillation

M = mass of the bowl and magnet = 0,030 kg
k = spring constant = 3 N/m
m = spring mass = 0,015 kg
L = length of the spring = length of the windings = 0,24 m

Procedure: The upper part of the induction coil is vertically positioned 5 cm below the lower end of the bowl. The function generator F12 is set to Sinus waveform. Set the drive frequency, then slowly turn the Amplitude-knob clock-wise until a satisfactory standing wave is observed - contractions and extractions of the spring. Then turn the Amplitude back to zero (fully CCW).

Measurement:

Now let the drive frequency run through the following interval

f(forced) between 1,30 Hz and 1,70 Hz
with 0,05 Hz steps.

The value of 1,45 Hz has been found as the resonance-frequency for the basic oscillation.

Analysis:

If drive force (amplification) is not too high, it is observed that exactly the drive frequency 1,47 Hz can pump enough energy into the spring pendulum.

The well-known formula $f = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$ gives with the values f = 1,60 Hz, and the slightly improved formula $f = \frac{1}{2\pi}\sqrt{\frac{k}{M + \frac{m}{3}}}$ gives the the value of 1,47 Hz.

These considerations and the measured value f = 1,45 Hz clearly shows that the results of the highly improved wave theory must be used with this oscillation apparatus.

The formula $f = \frac{1}{2\pi}\sqrt{\frac{k}{M + \frac{m}{3}}}$ is derived exactly as the 1st order approximation to the transcendental equation for the basic oscillation.

Verification:

Let the spring pendulum oscillate free - and measure the time of oscillation in the usual way, and calculate the frequency.

2. Experiment

Frequency-spectrum

M = mass of bowl and magnet = 0,030 kg
k = spring constant = 3 N/m
m = mass of spring = 0,015 kg
L = expanded length of spring = 0,24 m
that is the length of the windings

Procedure: The upper part of the induction coil is vertically adjusted to the position of the lower part of the bowl. First set the forced frequency - see the table on page 7 - then turn the Amplitude knob slowly clock-wise, until a satisfactory standing wave is observed - contractions and extractions of the spring windings. Then turn the Amplitude knob back to zero (CCW). New setup etc. etc.

Measurement:

Please look at page 7 containing measurements and calculations - but keep the procedure - never too high amplification. Often turn back the amplitude to zero, and then again slowly increase the amplitude.

Graph:

Please look at page 8.

3. Experiment

Calculation of the theoretical frequencies

With your own values for k, m and M, solve the transcendental equation

$$\cot(x) = ax$$

$$\text{with } a = \frac{M}{m} \text{ and } x = (2\pi f)\sqrt{\frac{k}{m}}$$

4. Experiment

Speed of longitudinal waves

1.Harmonic oscillation for closer study

The drive frequency is set to 7,6 Hz. Slowly increase the amplification, stop when the resonance is appropriate.

Observation: 2 knots on the spring and the lower knot is just above the bowl and the bowl itself is oscillating.

Measure: the distance from the upper knot to the lower knot - a value we have measured accurately to 0,223 m - important conclusion: not 24 cm, that is $\lambda/2 = 0{,}223\,m\ giving\ \lambda = 0{,}446\,m$

This result gives:

$$v_{(masured)} = f_{(measured)} \cdot \lambda_{(measured)} = 7{,}6\,Hz \cdot 0{,}446\,m = 3{,}39\,m/s$$

The theory gives:

$$v_{(theory)} = L \cdot \sqrt{\frac{k}{m}} = 0{,}24\,m \cdot \sqrt{\frac{3}{0{,}015}} = 3{,}39\,m/s$$

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10. Harmonic oscillation comments:

Here 11 knots are observed - that is 5 wave lengths - for the drive frequency 71,45 Hz. According to the refined wave theory for the spring pendulum, these 5 wave lengths are 0,237m = even here at the 10. Harmonic the lower knot is not located at the lowest spring winding.

Calculation:

$$v = 71{,}45\,Hz \cdot \left(\frac{0{,}237}{5}\right) m = 3{,}39\,m/s$$

Frequency-Spectrum for:
Frequency Pendulum

Oscillation condition	*Frequency calculated Spring fixed*	*Frequency measured Spring pendulum M=30g*	*Frequency calculated Spring free hanging with M=0g*
Basic oscillation	7,07	1,45	3,45
1. Harmonic	14,14	7,6	10,61
2. Harmonic	21,21	14,75	17,68
3. Harmonic	28,28	21,9	24,75
4. Harmonic	35,36	29,1	31,82
5. Harmonic	42,43	36,3	38,89
6. Harmonic	49,5	43,45	45,96
7. Harmonic	56,57	50,5	53,03
8. Harmonic	63,64	57,7	60,1
9. Harmonic	70,71	64,5	67,17
10. Harmonic	77,78	71,45	74,25

Calculation formula for the spring:

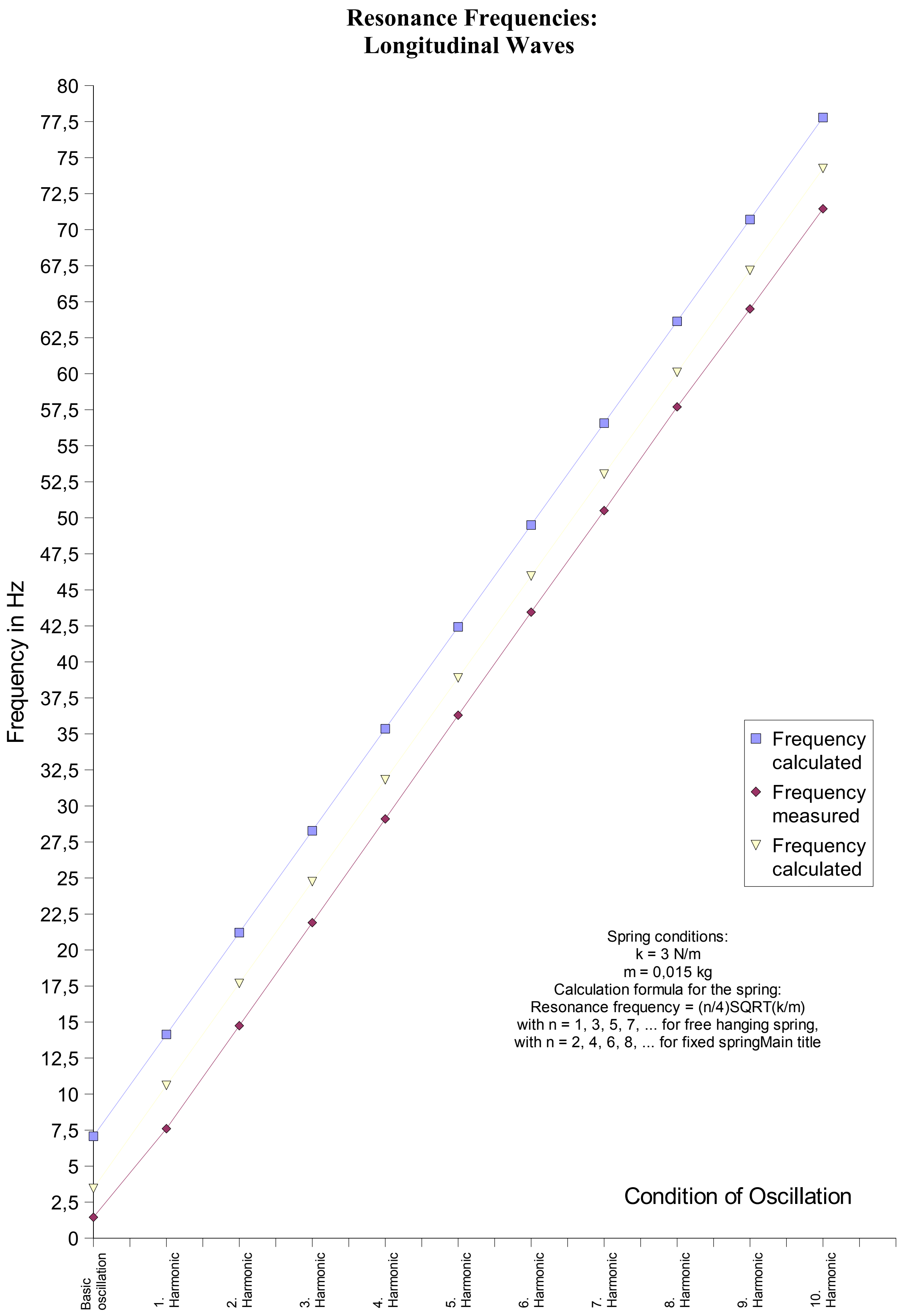
Resonance frequency = $\frac{n}{4}\sqrt{\frac{k}{m}}$

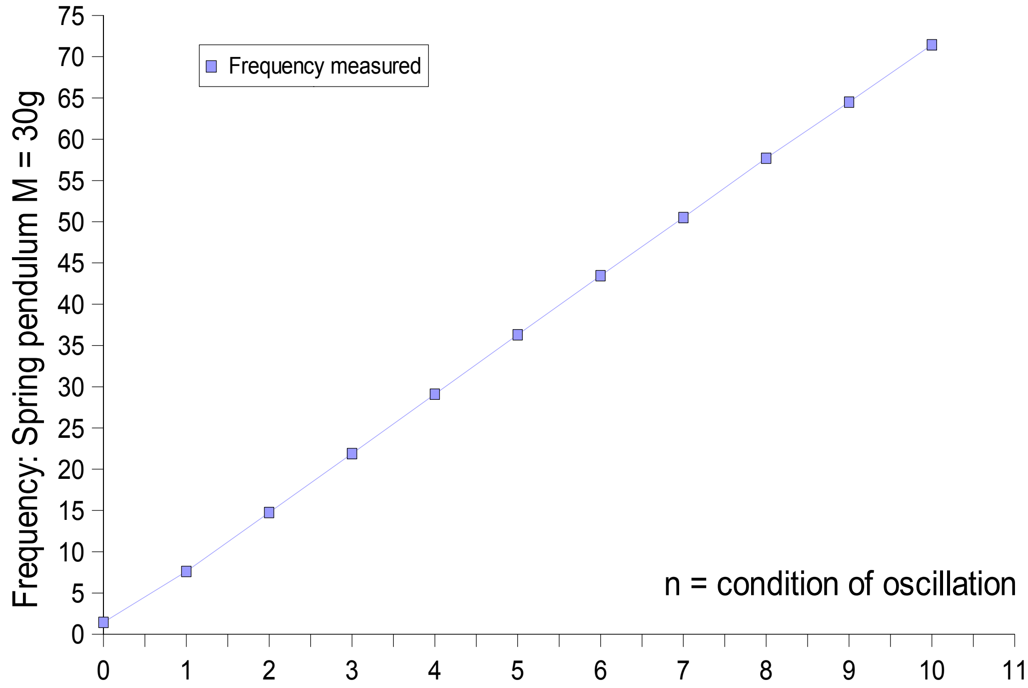
with **n = 2, 4, 6, 8, ...** for fixed spring
and with k = 3 N/m and m = 0,015 kg

Spring pendulum characteristics
k = 3 N/m
m = 0,015 kg
M = 0,030 kg (bowl + magnet)
L = 0,24 m

Calculation formula for the spring

Resonance frequency = $\frac{n}{4}\sqrt{\frac{k}{m}}$ with **n = 1, 3, 5, 7, ...** for free hanging spring, and with k = 3 N/m and m = 0,015 kg


Resonance Frequencies:
Longitudinal Waves
Frequency in Hz
80
77,5
75
72,5
70
67,5
65
62,5
60
57,5
55
52,5
50
47,5
45
42,5
40
37,5
35
32,5
30
27,5
25
22,5
20
17,5
15
12,5
10
7,5
5
2,5
0
Frequency calculated
Frequency measured
Frequency calculated
Spring conditions:
k = 3 N/m
m = 0,015 kg
Calculation formula for the spring:
Resonance frequency = (n/4)SQRT(k/m)
with n = 1, 3, 5, 7, ... for free hanging spring,
with n = 2, 4, 6, 8, ... for fixed springMain title
Condition of Oscillation
Basic oscillation
1. Harmonic
2. Harmonic
3. Harmonic
4. Harmonic
5. Harmonic
6. Harmonic
7. Harmonic
8. Harmonic
9. Harmonic
10. Harmonic



Best straight line fit: f = 7,075n + 0,884

Experimental control:

Best straight fit predicts the frequency of the 10. Harmonic to be 71,65 Hz – the experimental result gave 71,45.

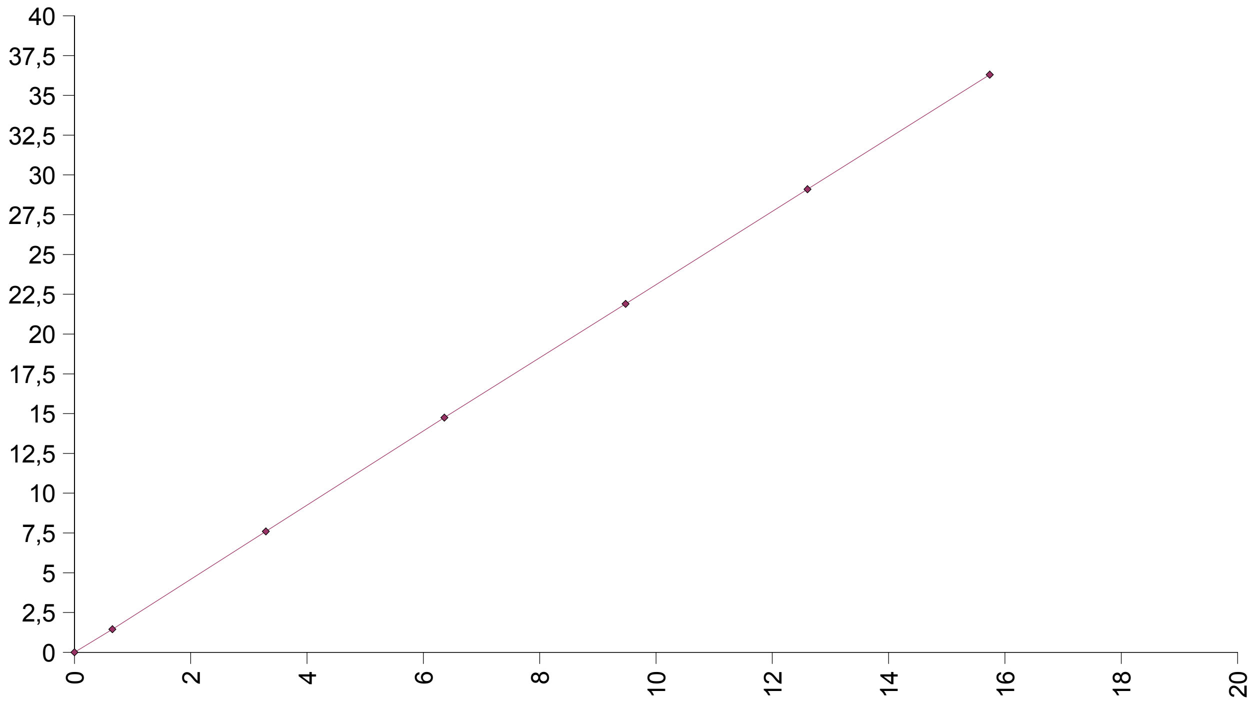
Try with the drive frequency 71,65 Hz – this frequency can not set the pendulum in the 10. Harmonic oscillation.

Conclusion: the straight line fit model is not usable.
The higher Harmonics becomes very sharp – narrow.

X calculated from cot(x) = 2x, that is M/m = 2
and the measured frequency f is compared to x

X calculated	*F measured in Hz*
0,6533	1,45
3,2923	7,6
6,3616	14,75
9,4775	21,9
12,6060	29,1
15,7397	36,3

Frequency: Spring pendulum M = 30 g:
the first 6 harmonics



Best straight line:
f = 2,309x – 0,005
that is K/m = 210
If m = 0,015kg, then k = 3,15 N/m
If k = 3 N/m, then m = 0,0143 kg

If the theoretical frequencies are precalculated, then k and m must be accurately known.

The Apparatus examined must have had k/m = 210.